A new airmass independent formulation for the Linke turbidity coefficient.

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Abstract

We propose a new formulation for the Linke turbidity coefficient with the objective of removing its dependence upon solar geometry. In the process we also develop two new simple clear sky models for global and direct normal irradiance.

Introduction

In a cloudless atmosphere, solar radiation is diffused by the permanent air molecules and scattered by the suspended solid and liquid particles. Information on the the quantity and the properties of these particles is needed to accurately estimate the clear sky radiation. The Linke turbidity coefficient T_L has been used since 1922 (Linke 1922) to quantifiy this information. However, T_L has the disadvantage to be dependent on the air mass. A number of authors have tried to circumvent this difficulty by different means. The most popular method was to normalize the measured values of T_L at air mass = 2 (Kasten 1988, Grenier 1994). Linke himself (1942) recognised the variation of T_L with air mass but had little success in introducing a new extinction coefficient based on an atmosphere of pure air containing 1 cm of water.

In the present paper, we develop a new formulation for the Linke turbidity factor, fully compatible with the original formulation at air mass 2, but roughly independent of the air mass. The first step was to modify existing direct (beam) and global irradiance clear sky models in order to better take into account the radiation's dependance with the altitude of the considered station and the solar geometry. The new turbidity factor is then derived by inversion of the beam clear sky radiation model.

The Linke turbidity coefficient

Linke (1922) proposed to express the total optical thickness of a cloudless atmosphere as the product of two terms, δ_{cda}, the optical thickness of a water- and aerosol-free atmosphere (clear and dry atmosphere), and the Linke turbidity coefficient T_L which represent the number of clean and dry atmospheres producing the observed extinction:

$$B_{nc} = I_o \cdot \exp(- \delta_{cda} \cdot T_L \cdot am)$$

where B_{nc} is the normal incidence direct irradiance and I_o the normal incidence extraterrestrial irradiance.

This definition depends on the theoretical value of δ_{cda} which is used to evaluate T_L. A careful examination of the definition of the terms δ_{cda} and T_L is helpful in getting a clear picture of Linke's formalism and the developments made since Linke first proposed it. Linke (1922) defined δ_{cda} as the integrated optical thickness of the terrestrial atmosphere free of clouds, water vapor and aerosols, which he computed from theoretical assumptions and apparently validated in a very pure, dry mountain atmosphere. He used the following formulation:

$$\delta_{cda} = 0.128 - 0.054 \cdot \log (am)$$

thus, T_L represents the number of clean dry atmospheres necessary to produce the observed attenuation, resulting from the additional and highly variable effects of water vapor and aerosols. Obviously, the smallest possible value of T_L at see level should be 1. Feussner and Dubois (1930) published a series of spectral data tables enabling the calculation of δ_{cda} where both molecular scattering and absorption by the stratospheric ozone layer are taken into account. Kasten (1980) fitted the following equation to these tables:

$$\delta_{cda} = (9.4 + 0.9 \cdot am)^{-1}$$

which is known as Kasten's pyrheliometric formula. In this widely used relation, absorption by the permanent atmospheric gases such as CO_2, O_2, N_2O, CO, etc. are not taken into account. The effect of these gases are therefore included in the term T_L, incorrectly contributing to atmospheric turbidity, as noted by Katz (1982) and confirmed by Kasten (1996). The dependence of δ_{cda} with air mass is a consequence of the strong dependence of Rayleigh scattering with the incident wavelength. As all the attenuation processes are dependant on wavelength, T_L is also dependent on air mass, although in a somewhat lesser manner than δ_{cda}. This variation of T_L for constant atmospheric clearness and the number of processes accounted for by T_L greatly hinders the practicality of Linke's formalism. This is illustrated on Figure 1 for the 10th of August, 1997 in Geneva. On the left, the Kasten modified Linke turbidity T_{LK} is plotted versus time of day. The right graph is a Langley plot for the afternoon of that day and for λ = 550 nm. The Langley plot shows that the quantity of aerosols can be considered as relatively constant for the afternoon, ground measurements of the atmospheric humidity and a similar plot for λ = 880 nm shows also a stability in the water vapor content of the atmosphere for the considered day, the morning Langley plots show a slightly different slope and confirm the lower value of T_{LK} in the morning. The dotted vertical lines indicate of an air mass = 2.

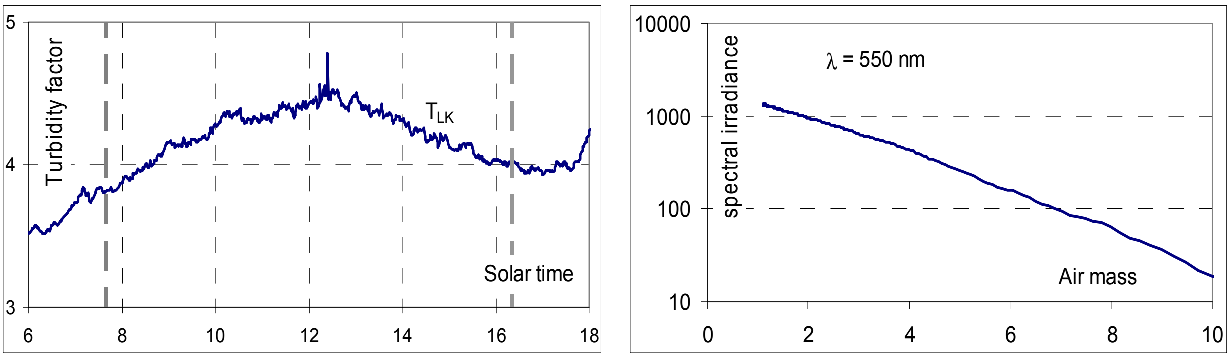

Figure 1 Evolution of the Kasten modified Linke turbidity coefficient during the 10th of August 1997 in Geneva (left), and for the same day, the afternoon Langley plot for λ = 550nm

Trying to improve the formulation, Louche (1986) and Grenier (1994, 1995) added absorption by the permanent gaseous constituents to the definition of δ_{cda} (these gases are considered uniformly mixed and invariable in both a clean dry atmosphere and a turbid atmosphere). Based on updated computed spectral data, Louche (1986) fitted a polynomial in fourth order of air mass to the optical thickness of a clean dry atmosphere. Grenier (1994, 1995), using a similar approach, added some minor changes to the spectral absorption and scattering equations yielding very similar values to Louche's (1986) relation:

$$\delta_{cda} = (6.5567 + 1.7513\ am - 0.1202\ am^2 + 0.0065\ am^3 - 0.00013\ am^4)^{-1}$$

The resulting values of δ_{cda} are higher (and hence the resulting values of T_L, will be smaller) than those obtained with Kasten pyrheliometric formula by as much as 25% for low values of air mass. Molineaux (1995) noted that Louche's and Grenier's expression for δ_{cda} become divergent respectively for air mass greater than 20 and 7. He adapted the coefficients of Linke's original expression to take into account the absorption by the permanents gases:

$$\delta_{cda} = 0.124 - 0.0656 \log (am)$$

Our approach in the determination of a new air mass-independent Linke turbidity coefficient is different. We based our definition on the Kasten-reviewed Linke turbidity T_{LK} at air mass 2 and considered it as the reference. For the beam and the global radiation components, we developped two empirical models that reproduce the observed shape of global and direct clear sky radiation measurements from seven environmentally distinct data banks. These models take into account the atmsopheric turbidity and the altitude of the considered location. We then inverted the direct clear sky radiation model to extract the turbidity coefficient and the corresponding optical thickness δ_{cda}. Linke's, Kasten's and Molineaux's optical depth are based on theoretical considerations, whereas our new formulation is derived from a large set of measurements. The behaviour of this clean and dry atmosphere optical thickness with the air mass is very different in comparison with the other definitions, but remains within the same limits as illustrated in Figure 2.

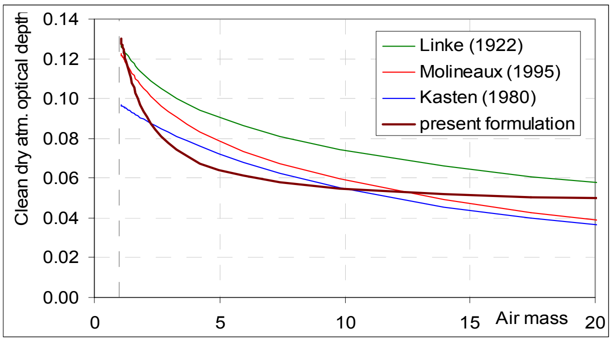

Figure 2 Clean and dry optical depth based on theoretical considerations defined by Linke, Molineaux and Kasten, and our new formulation derived from measurements.

Experimental data

We used data acquired at 7 stations with various latitudes, altitudes and climates to develop our models (ref. xx); they are the following:

- Albany (NY), latitude 42.7°, longitude -73.9°, altitude 100m
- Albuquerque (NM), latitude 35.1°, longitude -106.7°, altitude 1532m
- Burns (OR), latitude 43.5°, longitude -119.0°, altitude 1265m
- Burlington (KS), latitude 38.2°, longitude -95.6°, altitude 358m
- Eugene (OR), latitude 44.1°, longitude -123.1°, altitude 150m
- Geneva (Switzerland), latitude 46.2°, longitude 6.1°, altitude 420m
- Hermiston (OR), latitude 45.8°, longitude -119.4°, altitude 180m

Each of the data bank covers a full year of hourly data for the beam and the global radiation. The site of Albuquerque, Albany and Burlington are either part of the ARM program (Stokes and Schwartz, 1994) or apply the stringent ARM calibration, characterization and quality check procedure

Turbidity factor and normal incidence direct clear sky model

The basis of this clear sky radiation model is taken from Ineichen (1983). The model was initially developed from Geneva data. The formulation was an exponential attenuation fitted on the shape of the clear sky beam radiation measurements (from 1978 to 1982). The model did not take into account the turbidity of the atmosphere, but was based on a constant turbidity of T_L = 3. Introducing the altitude and the Linke turbidity factor at air mass = 2, a best fit on the data gives [1)]:

$$B_{ncI} = b\ I_o \exp(-0.09\ .\ am\ .\ (T_L - 1))$$

where the coefficients b is a function of altitude (f_{h1} is taken from Kasten (1984)):

$$b = 0.664 + 0.163/f_{h1} \qquad \text{where } f_{hl} = \exp(\text{altitude}/8000)$$

To illustrate the ability of the model to account for observed direct irradiance profiles, we represented on Figure 3 the beam clear sky index $K_b = B_n/B_{ncI}$ versus solar elevation for the station of Albuquerque. The points near the upper boundary represent clear sky conditions; the figure shows a relative stability of the upper boundary with solar elevation, hence show that the model adequately reproduces the clear sky air mass dependence of direct irradiance.

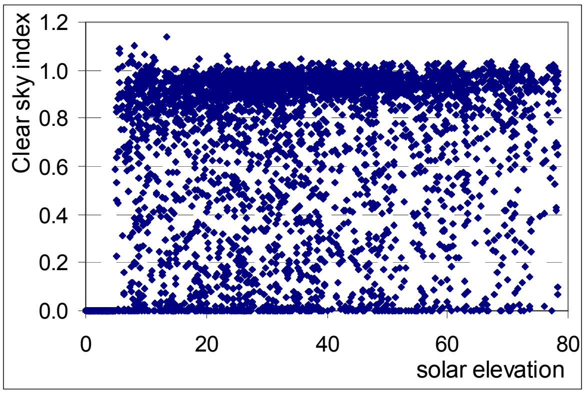

Figure 3 Beam clear sky index $K_b = B_n/B_{ncI}$ versus solar elevation. The upper boundary is representative of clear conditions.

We then inverted the expression and extracted the turbidity factor [2)]:

$$T_{LI} = [\ 11.1 * \ln(\ b\ I_o / B_{ncI})]/am\ ] + 1 \qquad \text{with } b = 0.664 + 0.163/f_{h1}$$

This new turbidity factor is represented on Figure 4 (left) for measurements taken the 10th of August 1997 in Geneva in comparison with the Kasten modified Linke turbidity factor

[1)] Further on in this paper, we describe a global radiation clear sky model. In order to remain coherent between the 3 radiation components, we apply a slight correction on the above model for very low solar elevations. The correction is given in the appendix.

[2)] In order to maintain coherence between the three global, direct and diffuse components, a small correction is added to this formulation as shown in appendix.

T_{LK}. The stability of the turbidity factor during the day is much better for the new coefficient. To further illustrate this point, for the same day, we made a simple radiative transfer simulation with SMARTS2 (Gueymard, 2001). We took the default values for O_2 and CO_2 absorption (taken from MODTRAN2 and SPECTRAL2 by Gueymard (2001)), 0.34 atm-cm for the ozone content, 0.0017 atm-cm for the NO_2, 1.3 for the Angström α coefficient, 2.37 pw-cm for the ground measured precipitable water vapor and a slight variation for the Angström β coefficient from 0.105 to 0.119 during the day (according to the T_L at air mass 2 in the morning and in the evening). The corresponding dotted lines are plotted on the right graph for both turbidity definition and are in good agreement with the measurements.

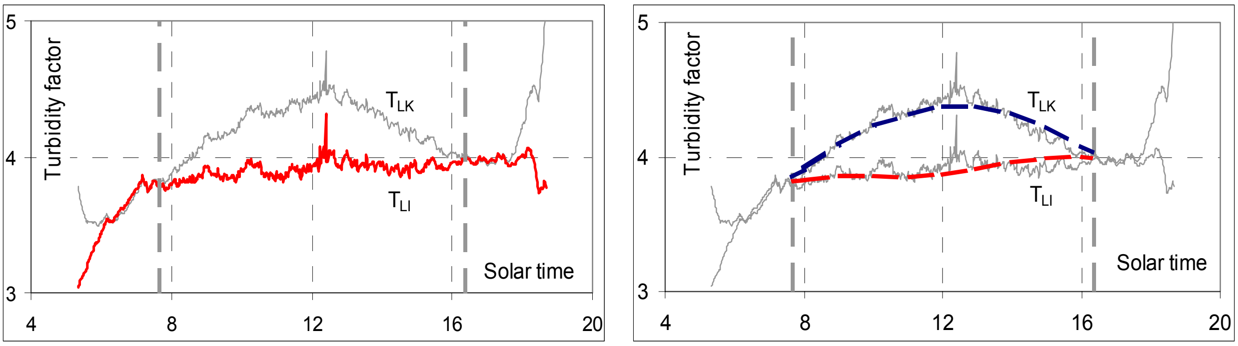

Figure 4 Evolution of the two turbidity coefficients during the 10th of August 1997 in Geneva (T_{LK} = original Linke coefficient, T_{LI} = new Linke coefficient). On the right graph, we superimposed the corresponding values from the SMARTS2 simulations.

Another possibility to illustrate the behaviour of the turbidity factor, is to represent hourly values versus solar altitude for all sky conditions. The lower boundary in the graphs represents clear sky conditions. On Figure's 5 left graph, the traditional T_{LK} is represented. The new formulation T_{LI} is shown on the right graph, both for data acquired at the station of Burns.

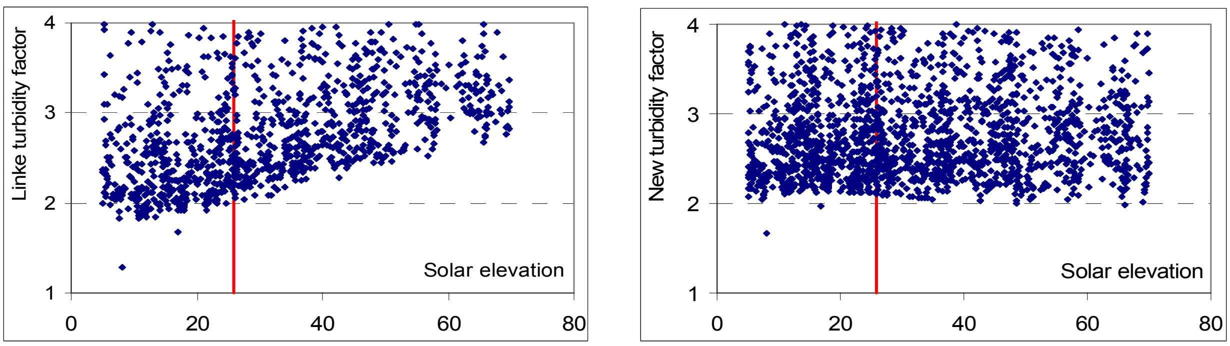

Figure 5 Hourly values of the two turbidity factors versus solar elevation in Burns, Oregon. The lower boundary is representative of winter clears sky conditions.

Global and diffuse clear sky models

Kasten (1984) proposed the following equation for the clear sky global radiation:

$$G_{hcK} = 0.84\ I_o \sin(h)\ .\ \exp(-0.027\ am\ (f_{h1}+f_{h2}\ (T_L-1)))$$

where G_{hcK} represents the global clear sky radiation reaching the ground on a horizontal surface, and f_{h1} and f_{h2} are coefficients (given in the nomenclature) that relate the altitude of the station with the altitude of the atmospheric interactions (Rayleigh and aerosols).

This formulation has the advantage to be adjustable for local/seasonal prevailing turbidity and site's elevation; it was developped on data from Hamburg. When applying it to our set of data, we observed a solar elevation- (or air mass-) and a slight altitude-dependance of the model. We plotted on Figure 6 the clear sky index $K_c = G_h/G_{hcK}$ versus the air mass on a logarithmic scale. The decrease with air mass of the upper boundary (clear conditions) on the left illustrates the air mass effect for data acquired at the station of Burns (altitude 1265m).

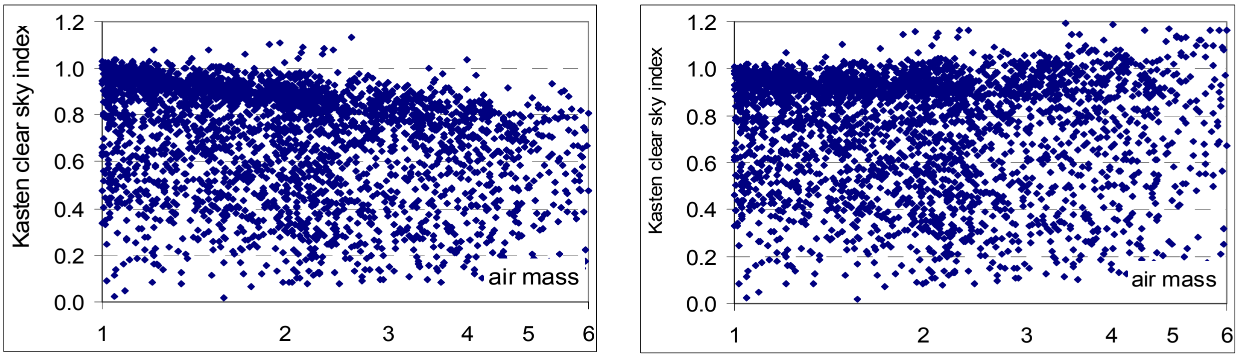

Figure 6 Global clear sky index $K_c = G_h/G_{hc}$ versus air mass. The decrease of the upper boundary on the left graph illustrates the model dependance with the air mass.

With the help of specific days under particular clear sky conditions extracted from the different data sets, we included in the Kasten model two altitude dependent coefficients:

$$G_{hcI} = a_1\ I_o \sin(h)\ .\ \exp(-a_2\ am\ (f_{h1}+f_{h2}\ (T_L-1)))$$

where:

$$a_1 = 5.09.10^{-5}\ .\ \text{altitude} + 0.868$$
$$a_2 = 3.92.10^{-5}\ .\ \text{altitude} + 0.0387$$

with the altitude expressed in meters. The corrected model is shown on the right in Figure 6. The diffuse clear sky radiation model is obtained by difference between the two components.

Conclusion

The Linke turbidity coefficient has ben widely used since 1922 to characterize the degree of transparency of the atmosphere. On the basis of measurements acquired at various geographic locations, altitudes and climates, we modified this turbidity coefficient to obtain a solar elevation-independant definition, fully compatible with the original.

In addition, we developped new formulations for clear sky global and direct irradiance taking into account the location's new Linke turbidity and altitude.

Nomenclature

G_h	global horizontal radiation	G_{hc}	clear sky global radiation
B_n	normal beam radiation	B_{nc}	clear sky beam radiation
D_h	diffuse horizontal radiation		
T_L	Linke turbidity coefficient		
T_{LK}	Linke turbidity coefficient corrected by Kasten		
T_{LI}	new Linke turbidity coefficient		
I_o	solar constant	am	optical air mass (Kasten 1989)
		h	solar elevation angle
f_{h1} =	exp(- altitude / 8000)	f_{h2} =	exp(- altitude / 1250)

Appendix

Beam clear sky model correction:

$$B_{ncI\,cor} = \min[\, B_{ncI} \,;\, G_{hc} * \{1 - (0.1 - 0.2 * \exp(-T_L))/(\,0.1 + 0.88/f_{h1})\} \,/\, \sin h]$$

the correction applies only on low solar elevation values.

Linke turbidity factor correction:

$$T_{LI\,cor} = T_{LI} - 0.25 * (2-T_{LI})^{0.5}$$

the correction applies for $T_{LI} < 2$

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